Key

Algebra 2 3.5 Zeroes of a Polynomial

Obj: Model and solve problems using the zeroes of a polynomial

Graphing from zeroes. Find intercepts set $y=0$, $x=0$ plot solve solve

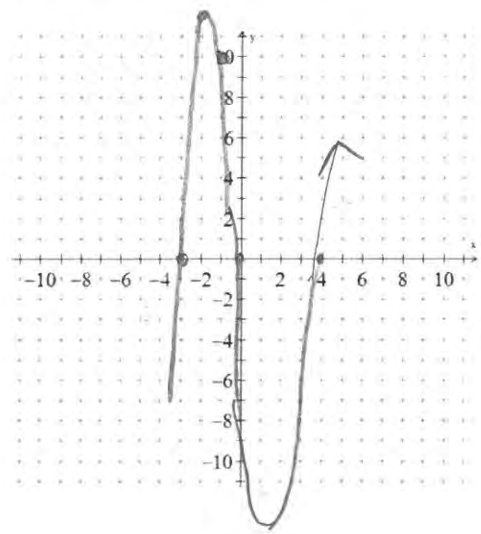
2, -6, 6

Example 1. Use the zeroes of the graph and its end behavior to sketch the polynomial.

$f(x) = x(x-4)(x+3)$ $\quad x=0 \quad x=4 \quad x=-3$

x	y
-1	10
-2	12
1	-12
2	-

Number line



$x \to -\infty \quad y \to -\infty \quad x \to \infty \quad y \to \infty$ ↙ ↗

change

You try. $f(x) = x^4 + x^3 - 4x^2 - 4x$

$f(0) = 0$

$0 = x^4 + x^3 - 4x^2 - 4x$

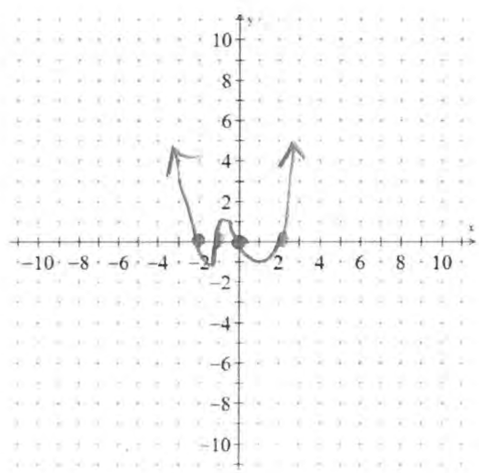
$x(x^3 + x^2 - 4x - 4)$

$x(x^2(x+1) - 4(x+1))$

$x(x^2 - 4)(x+1)$

$0 = x(x+2)(x-2)(x+1)$

$x=0 \quad x=-2 \quad x=2 \quad x=-1$



even +

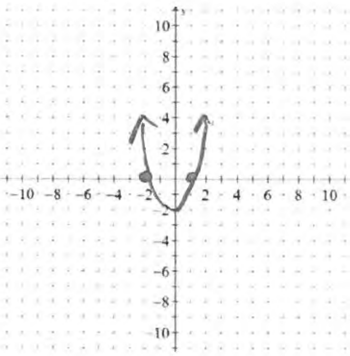
↖ ↗

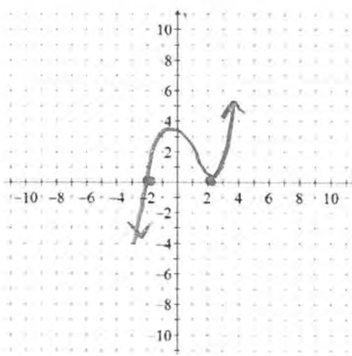
$x \to -\infty \quad f(x) = \infty$

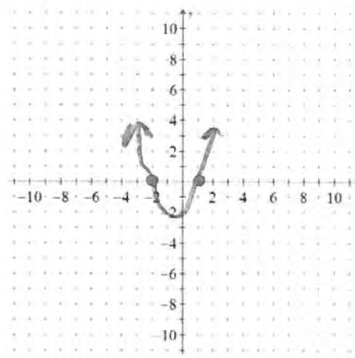
$x \to \infty \quad f(x) = \infty$

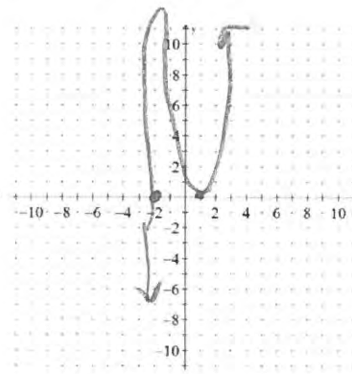
Example 2. **Multiplicity**. Graph the following on your Graph Calculator and sketch.

a. $f(x)=(x-1)(x+2)$ b. $f(x)=(x-1)^2(x+2)$ c. $f(x)=(x-1)^3(x+2)$ d. $f(x)=(x-1)^4(x+2)$









Zeroes:
$x=1$
$x=-2$

Zeroes:
$x=1$ 2 of them
$x=-2$

Zeroes:
$x=1$ 3 of them
$x=-2$

Zeroes:
$x=1$ 4 of them
$x=-2$

As the multiplicity of a zero increases what happens? what happens at the intercept changes

What pattern do you see? even # of repeats bounces off, $x=1$ $x=1$ odd number of repeats go thru $x=1$ $x=1$ $x=1$

Which create turning points?
even multiplicity (# of repeats)

Which are crossing points?
ODD # of multiplicity

Sketch a graph of the following function.

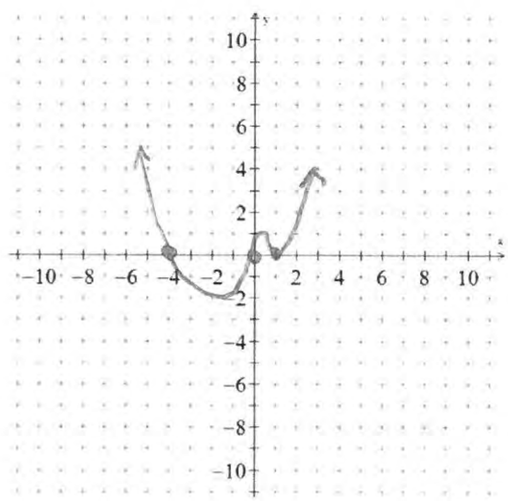
$f(x)=x(x-1)^4(x+4)$

$x\,x^4\,x$ so x^6 ↑ ↑

$x=0$ ↙ thru

$x=1$ 4 of them so bounces

$x=-4$ ↘ thru



Example 5 (you try #2). What are the solutions to $x^4 + 2x^2 = -x^3 - 2x$

or graph on Desmos

$x^4 + 2x^2 + x^3 + 2x = 0$

$x^4 + x^3 + 2x^2 + 2x = 0$

$x(x^3 + x^2 + 2x + 2) = 0$

$x(x^2(x+1) + 2(x+1)) = 0$

$x(x^2+2)(x+1) = 0$

$x=0 \quad x^2+2=0 \quad x=-1$

$x^2 = -2$

$x = \pm i\sqrt{2}$

~~**Example 6**. Solve a polynomial inequality by graphing. $x^3 - 16x < 0$~~

$2x^3 + 5x^2 - 3x = 3x^3 + 8x^2 + 1$

$0 = x^3 + 3x^2 + 3x + 1$

$x^2(x+3$

grouping Doesnt work

so graph on desmos and look

$x=-1$ is a zero

so

$$\begin{array}{r|rrrr} -1 & 1 & 3 & 3 & 1 \\ & \downarrow & -1 & -2 & -1 \\ \hline & 1 & 2 & 1 & \boxed{0} \end{array}$$

only solutions -1

$x^2 + 2x + 1 = 0$

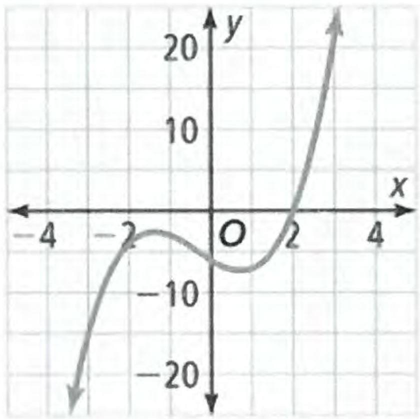
$x = \dfrac{-2 \pm \sqrt{4-4(1)}}{2} = \dfrac{-2 \pm \sqrt{0}}{2} = -1$

$(x+1)(x+1) = 0$

$x=-1 \quad x=-1$

so $x=-1$ multiplicity of 3

Example 3. Find real and complex zeroes. Use the graph to determine the real zeroes and then division to find any complex.



$f(x) = x^3 + x^2 - 3x - 6$

$x=2$ is a zero

Divide

$x-2 \overline{) x^3+x^2-3x-6}$ or

$$\begin{array}{r|rrrr} 2 & 1 & 1 & -3 & -6 \\ & \downarrow & 2 & 6 & 6 \\ \hline & 1 & 3 & 3 & \boxed{0} \end{array}$$

$x^2+3x+3=0$

solve w/ Quad Formula

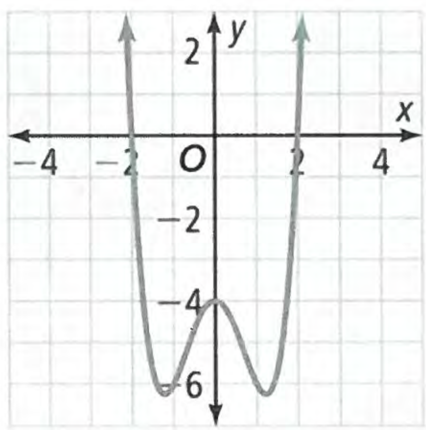
$a=1 \quad b=3 \quad c=3$

$x = \frac{-3 \pm \sqrt{9-4(1)(3)}}{2(1)}$

$x = \frac{-3 \pm \sqrt{-3}}{2}$

$\boxed{x = \frac{-3 \pm i\sqrt{3}}{2}}$

You try. $f(x) = x^4 - 3x^2 - 4$



$x=2$
$x=-2$

$x^4 + 0x^3 - 3x^2 + 0x - 4$

$$\begin{array}{r|rrrrr} 2 & 1 & 0 & -3 & 0 & -4 \\ & \downarrow & 2 & 4 & 2 & 2 \\ \hline & 1x^3 & 2x^2 & 1x & 2 & \boxed{0} \end{array}$$

now divide again

$$\begin{array}{r|rrrr} -2 & 1 & 2 & 1 & 2 \\ & \downarrow & -2 & 0 & -2 \\ \hline & 1x^2 & 0x & 1 & \boxed{0} \end{array}$$

$x^2+1=0$

$x^2=-1$

$x = \pm\sqrt{-1} = \boxed{\pm i}$